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Mapping letters to numbers: Potential mechanisms of literal symbol processing

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ABSTRACT

This study examined impermanent symbol-referent connections (e.g., $x = 5$) with literal symbols – important symbols for higher-level mathematics that may be difficult to process due to interference from pre-existing associations from literacy. We examined literal symbol processing at the group and individual levels, and executive functioning and symbol-referent mapping ability as potential cognitive mechanisms. Participants completed same-different judgments using numbers, literal symbols, and novel symbols; symbol-referent mapping; and executive function tasks. On average, participants' judgments with literal symbols took longer than with other symbols (i.e., a literal symbol processing cost). Individual differences in this cost were associated with variability in literal symbol mapping performance, but not with executive functioning. Results suggest the processing cost results from learning symbol-referent connections with symbols that have prior associations, and identifies initial symbol-referent mapping ability as a potential mechanism that contributes to this difficulty. Findings provide novel characterization of symbolic number processing relevant for higher-level mathematics.

1. Introduction

Mathematical ability is important for life success. It is related to physical and mental health (Garcia-Retamero, Andrade, Sharit, & Ruiz, 2015), risk assessment and medical decision making (Reyna & Brainerd, 2007), and may be more strongly related to earnings than literacy (Dougherty, 2003). One important aspect of mathematical ability is symbolic number processing, that is, connecting a symbol such as a digit to its numerical referent and working with that symbol in mathematical contexts. Research has established symbolic number processing as a consistent predictor of mathematics achievement (De Smedt, Noël, Gilmore, & Ansari, 2013; Schneider et al., 2017). Therefore, understanding the cognitive mechanisms that underlie symbolic number processing is of critical importance to support learners' development and adeptness with mathematics.

Numerical cognition research has focused on the cognitive mechanisms related to symbolic number processing for number words (Dehaene & Akhavein, 1995), digits (Defever, Sasanguie, Gebuis, & Reynvoet, 2011), rational numbers (Gabriel, Szűcs, & Content, 2013), and negative integers (Shaki & Petrusic, 2005). Common across these different symbolic formats and number types are permanent associations between symbols and their associated numerical magnitudes, such as how ONE, 1, ½, and −1 as symbols always refer to the same magnitude. Research on the link between symbolic number processing and mathematics achievement is specific to these permanent associations, usually with digits (De Smedt et al., 2013; Schneider et al., 2017). Further, current theories about the cognitive and neural mechanisms that facilitate symbolic number processing have focused on how a permanent association arises (Leibovich & Ansari, 2016; Reynvoet & Sasanguie, 2016). Therefore, most research and theories on symbolic number processing pertain only to symbols with permanent associations with numerical magnitude.

This focus excludes symbolic representations in mathematics beyond arithmetic. Intermediate and advanced mathematics incorporate alphabetic characters that represent numerical magnitudes (e.g., $x = 5$), which in this context are called literal symbols. In many cases, these symbols are variables (e.g., What happens to $1/x$ as x gets larger?), though literal symbols also represent numerical magnitudes in other contexts: to generalize arithmetic (e.g., $a + b = b + a$), as unknowns (e.g., $3y = 15$), and as parameters (e.g., $y = mx + b$), among other contexts (Usiskin, 1999). All these uses involve an impermanent connection between literal symbols and numerical magnitudes. For example, a literal symbol may be associated with the quantity 5, but this association is temporary. The association does not persist beyond a specific task or problem, and a different literal symbol may be associated with the quantity 5 in a different context. A theory of the

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development of mathematical cognition that goes beyond arithmetic must also include impermanent connections between symbols and their numerical magnitudes. Such theories can illuminate the well-documented pervasive difficulty that students have with literal symbols in mathematical contexts (Booth, 1999; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Küchemann, 1978; MacGregor & Stacey, 1997; McNeil et al., 2010; Philipp, 1992; Rosnick, 1999; Trigueros & Ursini, 2003). At present, little is known about the cognitive mechanisms that support impermanent symbol-referent connections with literal symbols, and accordingly, little is known about how learners process literal symbols that are used in more advanced mathematics.

Research related to impermanent symbol-referent connections has employed novel symbol learning paradigms that involve training participants to map (i.e., connect or associate) ordinal and/or magnitude information to unfamiliar novel symbols for use in subsequent tasks (e.g., comparison, number line estimation). Novel symbols are novel to participants and so lack any prior symbol-referent associations. Commonly-used novel symbols are Gibson figures (Gibson, Gibson, Pick, & Osser, 1962), letter-like symbols that, unlike letters, are new to participants. Prior studies that use novel symbol paradigms have used Gibson figures (Cohen Kadosh, Soskic, Iuculano, Kanai, & Walsh, 2010; Tzelgov, Yehene, Kotler, & Alon, 2000; Zhao et al., 2012), or study-specific novel symbols (Bennett, Inglis, & Gilmore, 2019; Lyons & Ansari, 2009; Merkley, Shimi, & Scerif, 2016). In these studies, participants either implicitly learn ordinal information about the novel symbols during training through several sessions of trial-and-error feedback (Cohen Kadosh et al., 2010; Tzelgov et al., 2000), learn ordinal information through sequential presentation of symbols in relative order (Merkley et al., 2016; Zhao et al., 2012), or learn magnitude information through pairing of symbols and non-symbolic dot arrays of 10 or greater (Bennett et al., 2019; Lyons & Ansari, 2009; Merkley et al., 2016; Zhao et al., 2012). Through subsequent tasks such as magnitude comparison, researchers have examined behavioral or neural markers associated with numerical magnitude processing (Lyons & Ansari, 2009; Tzelgov et al., 2000).

Yet, results of prior novel symbol training studies do not speak to the nature of literal symbol-referent mapping relevant for higher level mathematics. In mathematics, literal symbols are not mapped to numerical magnitudes through ordinal relationships with other literal symbols and are not associated with nonsymbolic dot arrays, but rather with other number symbols. Crucially, unlike novel symbols, literal symbols have prior referents - strong symbol-referent connections in the context of literacy - that may hinder mathematics understanding and performance (e.g.,Pollack, 2019; McNeil et al., 2010). Therefore, because of the uniqueness of literal symbols and their prevalence in mathematics, studies that focus on literal symbol processing are needed.

1.1. Literal symbol processing

Recent research has relied on symbolic number comparison tasks (e.g., which is larger, 4 or 6?) to characterize differences between processing literal symbols and other numbers (Pollack, 2019; Pollack, Leon Guerrero, & Star, 2016). Often considered a hallmark of numerical magnitude processing, such tasks elicit comparison or priming distance effects, in which performance is less efficient when comparison involves numbers that are closer together compared to farther apart (Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Moyer & Landauer, 1967; Reynvoet, de Smedt, & Van den Bussche, 2009; Van Opstal, Gevers, De Moor, & Verguts, 2008; Verguts & Van Opstal, 2005). In an initial study on differences between digit and literal symbol processing (Pollack et al., 2016), participants learned to associate literal symbols with a single numerical magnitude (e.g., $y = 9$) and then used the mapping in a subsequent numerical comparison task (e.g., which is larger, 4 or y?). The authors showed that digits produced distance effects, while literal symbols did not. These results provided initial evidence that numerical

Table 1
Literal and novel symbols associated with each digit.

	Digit			
	1	2	7	8
Literal symbol	Q	G	R	H
Novel symbol				

magnitude processing may differ between literal symbols and digits.

In an additional study, Pollack (2019) further examined literal symbol processing using a same-different task, in which participants determine whether two symbols (e.g., 2 – ONE) represent the same or different magnitudes. This task is well-suited to investigate literal symbol processing because performance on this task (including a same-different distance effect [SDDE]) reflects numerical magnitude processing (Dehaene & Akhavein, 1995; Van Opstal & Verguts, 2011), whereas number comparison tasks may not (Jou & Aldridge, 1999; Lyons, Nuerk, & Ansari, 2015; Paivio, 1975; Parkman, 1971; Van Opstal et al., 2008; Verguts & Van Opstal, 2005). In Pollack (2019), adolescents with beginning algebra knowledge first learned to map symbols to numerical magnitudes (e.g., $R = 7$) and then completed same-different tasks with digits and literal symbols (e.g., R and 8), and digits and novel symbols (i.e., Gibson figures; for an example, see Table 1). Results showed that both literal and novel symbols produced SDDEs, implying that participants processed numerical magnitudes associated with each symbol set. Results also showed for the first time that literal symbols produced a cognitive processing cost; that is, numerical magnitude judgments with literal symbols took longer on average than judgments with digits and with novel symbols. This finding suggested that pre-existing associations for literal symbols may contribute to longer processing times, compared to symbols without prior associations, because they interfere or compete with the new, impermanent associations.

1.2. Potential mechanisms

What cognitive mechanisms may contribute to the literal symbol processing cost? One candidate mechanism may be executive functioning. While prior studies (Pollack, 2019; Pollack et al., 2016) suggest that phonological working memory is unrelated to literal symbol processing, other executive functions such as visuo-spatial working memory and inhibitory control should be considered. Each contribute to numeracy development and mathematics performance (Blair, Knipe, & Gamson, 2008; Cragg & Gilmore, 2014; Li & Geary, 2013, 2017). Both visuo-spatial working memory and inhibitory control may mediate the relation between symbolic comparison with digits and mathematics performance (Price & Wilkey, 2017), suggesting a connection with basic symbolic number processing. These executive functions may also support literal symbol processing since literal symbols are well-known as letters of the alphabet. Because both numbers and letters have a spatial and linear ordering, it may be that visuo-spatial working memory facilitates symbolic same-different numerical judgments. Inhibitory control may also play a role, if the literal symbol processing cost is due to interference of automatic processing of literal symbols as letters of the alphabet (i.e., the automatic activation of letter identities and sounds).

A second candidate mechanism for a literal symbol processing cost could be the initial symbol-referent mapping (i.e., association) process that takes place during training. In literal symbol processing paradigms, learners map literal symbols to digits (e.g., $x = 5$) and then subsequently use that mapping to complete numerical judgments. Learners who are able to accurately, quickly, and reliably map a literal symbol to its associated referent during training may have smaller literal symbol processing costs.

In sum, processing literal symbols may be more demanding than processing digits or novel symbols because of impermanent associations with numerical magnitudes and pre-existing associations as letters of the alphabet. The result of these demands may be a literal symbol processing cost. However, little is known about this cost, including whether it may be a by-product of learners' inexperience with literal symbols in a numerical context, whether there are meaningful individual differences in this cost, and the potential cognitive mechanisms that contribute.

1.3. Present study

In the present study, we tested same-different numerical magnitude judgments with digits, literal symbols, and novel symbols in adults who had completed advanced mathematics courses. We chose same-different judgments because, as discussed above, they involve numerical magnitude processing, while comparison tasks may not (Van Opstal et al., 2008; Van Opstal & Verguts, 2011). By including same-different judgments with both literal and novel symbols, we are able to compare performance across two symbol sets that require impermanent associations with numerical magnitudes (unlike digits). In other words, judgments with novel symbols serve as a control task with the same task demands (e.g., learning symbol-magnitude associations, same-different tasks with non-digit symbols) as judgments with literal symbols. The two tasks vary only in symbol set; literal symbols have pre-existing associations from literacy, while novel symbols do not.

The present study had three aims. First, to examine whether a literal symbol processing cost (i.e., the difference between average response times for judgments with literal and novel symbols) exists in adults with extensive experience working with literal symbols in a mathematical context. Pollack (2019) showed a literal symbol processing cost for adolescents, which could have been an artifact of inexperience working with literal symbols. We hypothesized that average response times would be longer for literal symbol judgments than novel symbol judgments, since literal symbols have pre-existing associations that may interfere with processing, while novel symbols do not. The second aim was to examine for the first time the individual differences in the literal symbol processing cost. Even if this cost exists on average at the group level, is it present at the individual level? While behavioral signatures of symbolic number processing with digits are well-known at the group level (i.e., on average across participants; Ansari, 2008), recent research suggests these effects are unreliable at the individual level (i.e., on average across trials within an individual; Lyons et al., 2015). We considered this analysis exploratory. The third aim was to investigate two potential mechanisms that may relate to a literal symbol processing cost. We examined associations between literal symbol processing costs and individual differences in executive function measures and mapping performance, respectively. We hypothesized that greater executive function skills and mapping performance during training would be associated with a smaller literal symbol processing cost when making same-different judgments.

2. Materials and methods

2.1. Participants

Participants were 52 healthy native English-speaking students and young adults from 18 to 26 years old (M = 21.04, SD = 1.81; 77% female, 88% right handed) without diagnosed learning disabilities. All but one participant had completed at least a first year calculus course. We sought participants who had completed a calculus course as a marker of having extensive experience working with literal symbols. All participants gave written consent and received monetary compensation for participating. The study was approved by the university Institutional Review Board.

Four participants were excluded from the analyses. Due to a data collection error, training data for the literal symbols condition are missing for one participant and same-different judgment data for the literal symbols condition are missing for one participant. An additional participant performed at chance on one of the executive function measures (i.e., the Flanker task). One participant did not learn the associations during literal and novel symbols training and same-different judgments.

2.2. Procedure

Participation involved one 90-minute session. Participants began with five symbolic processing tasks: three same-different tasks and two training tasks, which were adopted from (Pollack, 2019). Symbolic processing tasks were created using OpenSesame 2.8.3 (Mathôt, Schreij, & Theeuwes, 2012) and presented using the OpenSesame Experiment Runtime Application on Google Nexus 7 tablets. Participants then completed two executive function tasks that measured working memory (i.e., Corsi span) and inhibition (i.e., Flanker task), respectively. Both tasks were administered using Eprime software (Psychology Software Tools, Inc. [E-Prime 2.0], 2012).

2.3. Symbolic processing tasks

To provide a baseline, participants began with a same-different task involving number symbols. Then, participants completed two sets of tasks that involved mapping symbols with numerical magnitudes and completing same-different judgments with the new symbols. The first symbol set (i.e., literal symbols, novel symbols) that participants worked with was counterbalanced.

2.3.1. Number symbols

The first same-different task used the digits 1, 2, 7, 8 and their number word equivalents (Van Opstal & Verguts, 2011). Cross-notation pairs eliminate visual matching that interferes with semantic processing of magnitude (Defever, Sasanguie, Vandewaetere, & Reynvoet, 2012). Same pairs (e.g., ONE – 1) had distance zero and Different pairs (e.g., 2 – EIGHT) were Near with distance of one, or Far with distances of five, six, or seven. As in (Van Opstal & Verguts, 2011), each block was comprised of 32 Same trials (i.e., eight pairs shown four times each), 16 Near trials (i.e., eight pairs shown twice), and 32 Far trials (i.e., 16 pairs shown twice), giving 80 trials per block. There were three blocks, yielding 240 trials total.

Trials began with a 500 ms fixation dot. Then, each number pair displayed on the screen until response. Participants determined whether each number pair represented the same or different magnitude. Participants held the tablet in landscape orientation and used their thumbs to touch the left side of the screen for same magnitudes and the right side of the screen for different magnitudes. Each trial ended with a 500 ms inter-stimulus interval (ISI). Panel A in Fig. 1 illustrates a sample trial.

2.3.2. Literal and novel symbols

Prior to completing same-different tasks with literal and novel symbols, participants completed a training during which they mapped numerical magnitudes with the symbol set (i.e., literal symbols or novel symbols). Table 1 shows the symbols and their numerical equivalents. Novel symbols were Gibson figures and their numerical equivalents as used in prior research on novel symbol learning (Tzelgov et al., 2000). Participants studied the four associations simultaneously for a minimum of 20 s. Then participants completed a training in which they viewed one of the symbols for 750 ms, recalled the associated numerical magnitude, and selected it from a choice screen. The position of the four numbers on the choice screen changed with every trial. The participant received feedback of “Correct!” or “Oops!” and saw the correct association for 750 ms. The trial ended with a 500 ms ISI. Participants completed eight practice trials and a minimum of 21 additional trials. Trials continued until participants reached 96 trials or at least 95% accuracy, whichever came first. Trials were in a pseudorandom order

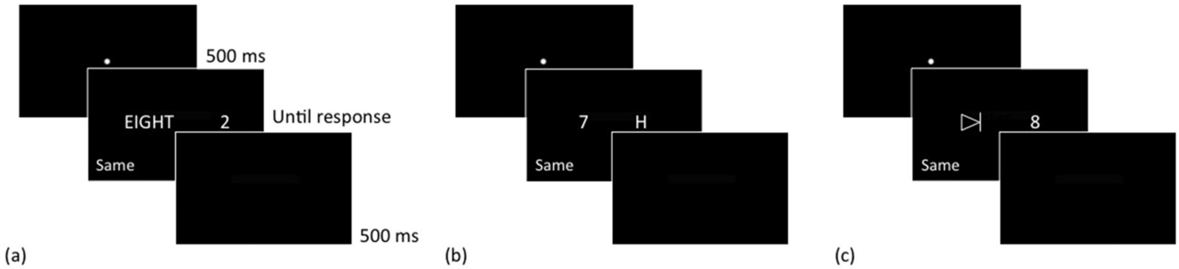

Fig. 1. Example trial and timing in the (a) number symbols condition, (b) literal symbols condition, and (c) novel symbols condition. On all trials, the words Same and Different were displayed in small text on opposite lower corners of the screen.

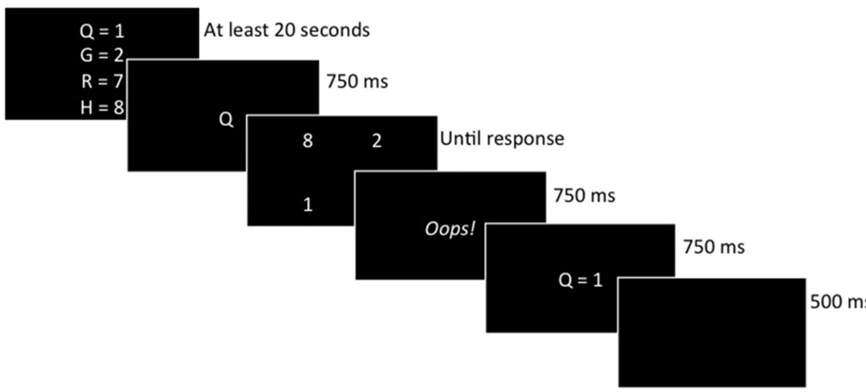

Fig. 2. Example of training task with literal symbols. Participants began by memorizing four associations. During training, participants saw a symbol, selected the associated number, received feedback, and saw the correct association.

that was the same for all participants, to ensure that all four associations were tested approximately equally. Fig. 2 shows an example of the training task in the literal symbols condition.

After training, participants completed same-different judgments in which the number words from the numbers-only condition were replaced with the literal or novel symbols (e.g., $H – 1$). Half of the trials had the new symbol on the left and the other half had the new symbol on the right. Other aspects of the same-different judgment were identical to the number symbols condition. Panels B and C in Fig. 1 show examples of trials and timing for the literal and novel symbols conditions.

2.4. Executive function measures

To assess working memory, participants completed the backwards version of the Corsi block tapping test (Corsi, 1972). Participants viewed nine gray squares on a black background. For each trial, a subset of the squares turned red sequentially and participants were asked to touch the squares in the reverse order of presentation. The task began with two practice trials of length 2. There were two trials at each length from 2 to 8 yielding 16 total trials. Participants advanced to length $n + 1$ if one of the trials at length n was recalled correctly. The task terminated if both trials of the same length were recalled incorrectly. Participants' Corsi span was the highest length in which at least one trial was recalled correctly.

To measure inhibition, we used a modified flanker task. This task measures participants' ability to suppress distracting information without involving symbol processing common to the same-different task (i.e., processing digits and alphabetic characters, as in a Stroop task). Participants saw five arrows on the screen and indicated via keyboard button press whether the center arrow pointed left or right, as quickly as possible without sacrificing accuracy. In congruent trials, all five arrows pointed in the same direction. For incongruent trials, the center arrow pointed opposite of the four flanking arrows. To prevent constant fixation on the center of the screen, the set of five arrows displayed to the left or right of center. To orient participants' attention, a trial began with a box outlining where the set of arrows would display, for 200 ms. The arrows displayed for 1000 ms, followed by a 2000 ms ISI with a blank screen. There were four practice trials and 80 experimental trials, counterbalanced for congruency, direction, and display location. Accuracy and response time were recorded for each trial.

2.5. Analytic approach

2.5.1. Same-different distance effects

As in prior research (Pollack, 2019; Defever et al., 2012; Sasanguie, Defever, Van den Bussche, & Reynvoet, 2011; Smets, Gebuis, & Reynvoet, 2013), ‘Same trials’ were not analyzed because the SDDE only manifests between Different trials of near and far distances. Error rates and median response times for correct trials were calculated for each participant. For the flanker task, error rate and median response time on correct trials were calculated for each participant, separately for congruent and incongruent trials. Measures of inhibition were calculated as the difference between incongruent and congruent trials, separately for accuracy and response time. We used 3 × 2 repeated-measures analysis of variance with factors of symbol set (i.e., Number, Literal symbols, Novel symbols) and distance (i.e., Near, Far). Due to low error rates (see Table 2 for error rate by symbol set), we focus the analyses on response time. For completeness, we provide an analysis of error rates in the appendix.

2.5.2. Power analysis for literal symbol processing cost

There are few studies on the literal symbol processing cost. However, we conducted an a priori power analysis based on its prior estimation (Pollack, 2019). A power analysis using G*Power (Faul, Erdfelder, Buchner, & Lang, 2009; Faul, Erdfelder, Lang, & Buchner, 2007) suggested that with $\alpha = 0.05$ and 80% power, a sample size of 47 was needed to detect differences between two dependent means of 0.42 standard deviations (see Pollack, 20199). Further, a sample size of 48

Table 2
Descriptive statistics for same-different judgments for the three symbol sets. Descriptive statistics for Same trials are included for completeness, but are not a part of the SDDE analysis. Response times are averages of participant-level median response times, rounded to the nearest ms ($n = 48$).

Measure		Mean	SD	Min	Max
Response time					
Number	Near	732	109	563	1038
	Far	710	122	519	1232
	Same	698	101	533	1039
	Overall	710	107	534	1107
Literal symbols	Near	907	291	615	2438
	Far	890	315	609	2673
	Same	822	204	620	1834
	Overall	856	289	506	2064
Novel symbols	Near	798	116	618	1334
	Far	791	126	608	1313
	Same	772	92	633	1011
	Overall	783	102	625	1164
Error rate (% error)					
Number	Near	2.21	3.17	0.00	14.58
	Far	0.91	1.23	0.00	4.17
	Same	4.75	3.44	0.00	14.58
	Overall	2.71	1.97	0.42	10.00
Literal symbols	Near	2.99	3.33	0.00	14.58
	Far	1.54	2.02	0.00	8.33
	Same	5.73	4.18	0.00	19.79
	Overall	3.51	2.36	0.42	10.83
Novel symbols	Near	1.30	1.95	0.00	6.25
	Far	1.45	1.88	0.00	9.38
	Same	5.77	4.58	0.00	20.83
	Overall	3.15	2.32	0.00	10.83

exceeds sample sizes from related studies that have estimated performance differences on the same-different task across notations using analysis of variance methods (e.g., Ganor-Stern & Tzelgov, 2008; Van Opstal & Verguts, 2011). Together, this suggests that the current sample size is adequate to detect differences between literal and novel symbol processing. However, the present study provides the first estimates of the correlation between the literal symbol processing cost and its potential mechanisms (i.e., executive function and mapping measures), precluding an a priori power analysis for those analyses.

2.5.3. Individual processing costs

The above analyses will estimate a literal symbol processing cost at the group level, but not at the individual level. We used Lyons et al.'s (2015) approach of fitting individual regression models with trial-level data from numerical judgments, in order to estimate here the average response time difference between literal and novel symbol judgments, controlling for distance. This approach treats trials for each participant as separate observations and estimates a literal symbol processing cost for each participant. We use this approach rather than calculating differences scores for two main reasons. First, it provides a more precise estimate of the processing cost between literal and novel symbols by controlling for distance. Second, unlike with difference scores, this method incorporates the precision with which the processing cost is estimated. Therefore, this approach provides individual level estimates of the magnitude of the difference between literal and novel symbol processing (as difference scores also provide) and a standard error (which difference scores alone do not provide), which together are an indicator of whether the processing cost is statistically significant or is likely due to sampling idiosyncrasy.

2.5.4. Mapping measures

To ensure a mapping between the symbols and their referents, participants were required to achieve a high level of accuracy on the training tasks. Consequently, this essentially eliminated variability in accuracy rates. Therefore, we operationalize mapping performance in terms of average response time (i.e., a measure of speed) and variability in response time (i.e., a measure of reliability), and examine associations between these measures and literal symbol processing costs.

3. Results

3.1. A literal symbol processing cost

3.1.1. Descriptive statistics

Table 2 presents the error rate and response times for same-different judgments for the three symbol sets. As shown in Table 2, the sample data showed a pattern of SDDEs for response time, and for numbers and literal symbols for error rate. The sample data also showed longer response times, higher error rates, and greater variability for literal symbols compared to novel symbols and number symbols.

Average Corsi span was 6.27 (SD = 1.20); scores ranged from 3 to 8. Accuracy on the flanker task was 97.81% (SD = 5.45) and ranged from 70 to 100%. Average median response time was 572 ms (SD = 126). Average inhibition scores for response times were 80.23 ms (SD = 46.73) and for error rates were 1.56 (SD = 3.21).

3.1.2. Response time

For response time, Mauchly's test showed a violation of the sphericity assumption for symbol set ($\chi_2(df = 2) = 58.029$, $p < .0001$) and the interaction of symbol set and distance ($\chi_2(df = 2) = 11.304$, $p = .004$). Degrees of freedom were adjusted using the Greenhouse-Geisser correction ($\varepsilon = 0.58$ and $\varepsilon = 0.82$, respectively). There was a statistically significant main effect of symbol set [$F(1.17, 54.75) = 20.22$, $p < .0001$, $\eta_p^2 = 0.301$]. Bonferroni-adjusted pairwise comparisons showed that response time was about 177 ms faster for numbers than literal symbols ($t = -4.93$, $p < .0001$, 95% CI [−267, −88]) and about 73 ms faster for numbers than novel symbols ($t = -5.75$, $p < .0001$, 95% CI [−104, −41]). These results illustrate the expected faster response time for the number condition compared to the other two conditions, which required new symbol-referent mapping. Crucially, there was a literal symbol processing cost, in which response time was 104 ms longer for literal symbol trials than novel symbol trials ($p = .003$, 95% CI [30, 179]). There was a statistically significant main effect of distance [$F(1, 47) = 5.32$, $p = .03$, $\eta_p^2 = 0.102$] in which responses to Near trials took about 15 ms longer than Far trials, on average ($t = 3.48$, $p = .03$, 95% CI [1.90, 27.83]). The interaction of symbol set and distance was not statistically significant [$F(1.64, 77.18) = 0.96$, $p = .47$, $\eta_p^2 = 0.015$]. Fig. 3 displays the estimated means by symbol set.

3.2. A literal symbol processing cost at the individual level

Of the 48 participants included in the analysis, 38 (i.e., 79%) of them showed statistically significant differences in response time for same-different judgments between literal and novel symbols, on

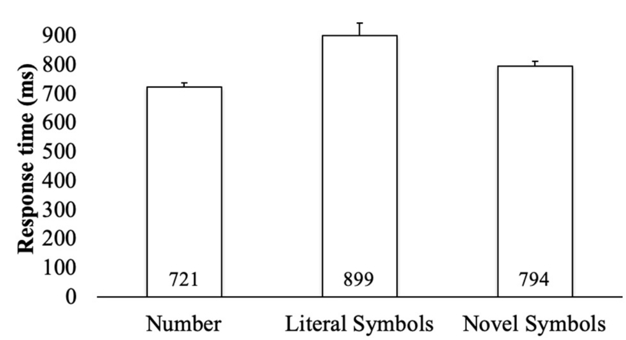

Fig. 3. Estimated response time by symbol set. Error bars represent standard errors ($n = 48$).

average, controlling for distance, and 10 (i.e., 21%) did not. Sixty percent of participants (29 of 48) showed a statistically significant literal symbol processing cost, whereas 19% (9 of 48) showed a novel symbol processing cost (i.e., a negative parameter estimate). Differences in response time between literal and novel symbol processing ranged from −229 ms (slower response time for novel symbols) to 2033 ms. These results suggest that, while not universal, the majority of participants showed statistically significant differences between processing literal and novel symbols, and when they did, they most often (29 of 38) exhibited a literal symbol processing cost, regardless of the numerical distance between the two symbols being compared.

3.3. Potential mechanisms

3.3.1. Lack of evidence for a relation to executive functions

We investigated executive functioning as a potential cognitive mechanism contributing to the literal symbol processing cost. To do so, we correlated the magnitude of the processing cost (i.e., the difference in response time between literal and novel symbol judgments) with measures of visuo-spatial working memory and inhibition. The relation between the literal symbol processing cost and Corsi span was not statistically significant ($r(46) = 0.26$, $p = .08$). Similarly, the relationship between the literal symbol processing cost and inhibition was not statistically significant for inhibition measured using error rate ($r(46) = -0.07$, $p = .64$) or response time ($r(46) = 0.01$, $p = .95$).

To check that relations between the literal symbol processing cost and executive function ability were not attenuated due to including the full sample, we estimated the same correlations using only participants with a statistically significant literal symbol processing cost. Results were unchanged. There was not a statistically significant correlation for Corsi span ($r(36) = 0.31$, $p = .06$). Because there was one participant with a relatively large literal symbol processing cost, we conducted a sensitivity analysis to determine whether this participant may be influencing the correlation. With this participant excluded, the correlation was lower and the *p*-value was higher ($r(35) = 0.22$, $p = .20$), supporting the lack of a statistically significant correlation. Results were also unchanged for correlations between the processing cost with inhibition measured with error rate ($r(36) = -0.08$, $p = .65$) and response time ($r(36) = -0.02$, $p = .92$). Taken together, these results do not provide evidence for executive functions as a mechanism underlying literal symbol processing.

3.3.2. Evidence for a relation to mapping performance

To test for differences in the quality of symbol-referent mapping across literal and novel symbols during training, we first compared participants' average number of training trials (since task completion was contingent on accuracy) and median response times. The average accuracy rate for literal symbol training was 98.29% (SD = 2.57) and 99.60% (SD = 1.33) for novel symbols. For literal symbols, participants completed an average of 23 non-practice trials (M = 23.4, SD = 11.44) with a range of 21–96 trials. For novel symbols, all participants completed 21 non-practice trials (SD = 0). A paired-samples *t*-test showed that on average, the number of trials did not differ between the two training tasks ($t(47) = 1.45$, $p = .15$). For literal symbol training, participants' average median response time was 634 ms (SD = 171 ms) and for novel symbol training, average median response time was 572 ms (SD = 92 ms). A paired-samples *t*-test showed that mapping took longer for literal symbols compared to novel symbols, on average ($t(47) = 2.87$, $p = .006$). A variance comparison test showed greater variation in participants' response times for literal symbol training compared to novel symbol training, on average ($F(47, 47) = 3.48$, $p < .0001$). These results suggest that, at the group level, the mapping between a literal symbol and its numerical referent may be less efficient than the same mapping involving novel symbols, even when that mapping is achieved with high accuracy and with similar levels of exposure.

We additionally examined the association between the magnitude of the literal symbol processing cost and individual differences in mapping performance for literal symbols, in terms of median response time (i.e., a measure of speed) and standard deviation (i.e., a measure of reliability). Longer median response time during literal symbols training was associated with a larger processing cost ($r(46) = 0.61$, $p < .0001$). However, this relationship was sensitive to the presence of one participant with relatively larger values and does not persist when that participant is excluded ($r(45) = 0.19$, $p = .20$). The pattern of results was similar for participants who show a reliable processing cost. There was a correlation between median response time during training and processing cost ($r(36) = 0.68$, $p < .0001$) that was driven by the presence of the same relatively large value ($r(35) = 0.21$, $p < .20$), suggesting there was not a meaningful relation between median response time and processing cost. For variability of the symbol-referent mapping, a larger response time standard deviation during literal symbols training was associated with larger processing costs ($r(46) = 0.44$, $p = .002$). In contrast, there was no statistically significant relationship between standard deviation in response times during novel symbols training and processing cost ($r(46) = 0.17$, $p = .25$). Additional analyses of the relation between processing cost and mapping performance with only participants who had a statistically significant processing cost produced the same results. Taken together, these results suggest that literal symbol-referent mapping variability relates to the magnitude of the literal symbol processing cost.

3.3.3. Comparing correlations across working memory and mapping performance

As shown above, the correlation between Corsi span and the literal symbol processing cost ($r(46) = 0.26$, $p = .08$) suggested that visuo-spatial working memory was unrelated to literal symbol processing. However, since the result was close to the $p = .05$ threshold, a conclusion of no relation may be too strong. To further examine visuo-spatial working memory as a candidate mechanism, we conducted a test of the difference between two dependent correlations (Lee & Preacher, 2013) to compare the magnitude of the above correlation with the magnitude of the correlation of the processing cost and mapping variability during training ($r(46) = 0.44$, $p = .002$). Results suggest that the correlation magnitudes are not statistically significantly different ($z = -0.91$, $p = .36$). We interpret these results cautiously, however, since there may not be sufficient power to detect a difference between correlations. While this does not provide strong evidence for a relation between visuo-spatial working memory and the literal symbol processing cost, visuo-spatial working memory is of theoretical interest for literal symbol processing, and so may need further consideration.

4. Discussion

4.1. Symbol processing and performance across symbol sets

In the present study, we examined a literal symbol processing cost at the group and individual levels for adults with a high level of mathematical knowledge, and whether working memory, inhibition, and mapping performance contributed to that cost. To do so, participants completed same-different numerical judgments with three different symbol sets, training tasks that required mapping symbols to numerical referents with a high degree of accuracy, and executive function measures of working memory and inhibition.

At the group level, participants showed a response time SDDE for each symbol set, in which comparisons on Near trials took longer on average than comparisons on Far trials. The presence of SDDEs across symbol sets is in line with prior studies that utilize symbolic same-different numerical judgments (Defever et al., 2012; Dehaene & Akhavein, 1995; Pollack, 2019; Sasanguie et al., 2011; Van Opstal & Verguts, 2011) and shows that all symbol sets displayed signatures of numerical magnitude processing.

4.2. Literal symbol processing and performance

Importantly, average response times differed across symbol sets, showing support for a literal symbol processing cost at the group level. Same-different judgments with numbers took less time, on average, than with both literal and novel symbols, and judgments with literal symbols took longer than with novel symbols. Longer average response times for literal and novel symbols compared to numbers may reflect common cognitive processes that are necessary for retrieving and applying novel symbol-referent connections. However, a longer response time for judgments with literal symbols compared to novel symbols suggests additional processing demands for literal symbols beyond novelty. This in turn suggests that there is a characteristic of literal symbols that makes them difficult per se, such as their prior associations from literacy. Learners have strong representations for literal symbols as letters of the alphabet (e.g., as known symbols that represent sounds and form words) prior to seeing them in a mathematical context. Indeed, prior to mathematics instruction, some young learners assume that literal symbols represent their ordinal position in the alphabet (e.g., A = 1, B = 2); however, this misconception resolves early-on with instruction (Stacey & MacGregor, 1999) and is highly unlikely to be at play here.

The present study is the first to examine the literal symbol processing cost at the individual level. Seventy-nine percent of participants showed statistically significant differences between literal and novel symbol processing. This finding supports the notion that symbol set affects performance on numerical judgment tasks and identifies this effect as a candidate mechanism that may relate to individual differences in mathematical performance on other tasks. While a majority of participants showed a literal symbol processing cost, nine participants showed a statistically significant novel symbol processing cost. For some learners, literal symbols may actually facilitate same-different judgments. While student difficulties with literal symbols are pervasive and persistent (Küchemann, 1978; McNeil et al., 2010; Stacey & MacGregor, 1999; Trigueros & Ursini, 2003), not all students struggle with the literal symbol concept. Perhaps some learners' difficulties with literal symbols stem from pre-existing associations from literacy, while learners who are adept at literal symbol processing utilize the symbols' familiarity to aid problem solving. An open question is whether those who exhibit a literal symbol processing cost and those who show a novel symbol processing cost utilize different strategies during same-different judgments. Research on arithmetic suggests that strategy choice, rather than problem features, are associated with different neurocognitive mechanisms (e.g., Polspoel, Peters, Vandermosten, & De Smedt, 2017; Tschentscher & Hauk, 2014). Perhaps an analogous explanation applies here. Further, if literal symbol processing impedes or facilitates performance for individual learners, as the present study suggests, there may be a relation between literal symbol processing costs and performance on more authentic mathematics tasks, particularly in the beginning stages of algebra learning when learners are first exposed to literal symbols.

One could argue, however, that the literal symbol processing cost was a consequence of more difficult symbol-referent associations for literal symbols, and thus produced longer response times, because the pairings did not preserve the linear ordering across digits and literal symbols (i.e., G, H, Q, R with 1, 2, 7, and 8 respectively). Rather, symbol-referent associations were randomly assigned, which resulted in a numerical magnitude ordering for literal symbols that did not preserve the linear ordering of the alphabet (i.e., Q, G, R, H with 1, 2, 7, and 8). However, increased difficulty due to incongruent pairings based on linear ordering seems unlikely given that prior research has shown that the linear ordering of the alphabet does not affect performance on the same-different task (Van Opstal & Verguts, 2011). In addition, a congruent symbol-referent mapping for literal symbols was purposely avoided to disallow strategies that relied on ordinal reasoning rather than associating literal symbols with numerical magnitudes. Though unlikely, if ordinal positions of letters in the alphabet were to affect numerical magnitude processing, this would suggest one mechanism by which literal symbols are more difficult to work with in numerical contexts due to their pre-existing associations from literacy (i.e., ordinal representations). Such hypotheses could be probed explicitly in future studies by contrasting performance with symbol-quantity mappings that preserve and do not preserve the ordinality of the alphabet.

The presence and magnitude (i.e., 104 ms) of the literal symbol processing cost in adults who have taken higher-level mathematics courses aligns with prior research that found a similar cost in adolescents (Pollack, 2019). This suggests that a literal symbol processing cost does not result from inadequate exposure to literal symbols in a numerical context and that difficulty processing these symbols during numerical magnitude judgments is also present for those with a high level of mathematical knowledge. In sum, results of the present study reinforce the notion that on average, and for the majority of individual participants, there is an inherent difficulty in processing literal symbols in a numerical context. Even for very simple numerical tasks, working with these symbols is more difficult than working with other symbols that represent numerical magnitude. That these symbols are difficult to interpret per se may be concerning, considering the ubiquity of literal symbol use in intermediate and higher-level mathematics.

4.3. Potential cognitive mechanisms

The present study also contributes novel insights related to the potential cognitive mechanisms that underlie the literal symbol processing cost. We hypothesized that participants with greater executive functioning would tend to have smaller literal symbol processing costs. However, results did not support this hypothesis. Measures of non-numeric visuo-spatial working memory and inhibition were each uncorrelated with processing cost magnitude. These results add to findings from prior studies that did not find a relationship between literal symbol processing and phonological working memory measured with a mathematical operation span task (Pollack et al., 2016) and backward digit span task (Pollack, 2019). While the relation between visuo-spatial working memory and the literal symbol processing cost would benefit from additional examination, together these studies do not provide strong evidence that literal symbol processing depends on visuo-spatial or phonological working memory. These findings stand in contrast to the role of verbal and spatial working memory in spatial-numerical associations involved in some symbolic number processing tasks, such as parity judgment and number comparison (Fias & van Dijck, 2016; Fias, van Dijck, & Gevers, 2011; van Dijck & Fias, 2011; van Dijck, Gevers, & Fias, 2009). Further, results suggest that the ability to suppress distracting information is unrelated to the literal symbol processing cost. However, susceptibility to proactive interference, in which prior learning impedes learning of new information (e.g., Keppel & Underwood, 1962; Underwood, 1957), may play a role. If so, one may speculate that a larger literal symbol processing cost may be associated with greater susceptibility to proactive interference. Lastly, it may be that executive functions could play a role in switching between different referents for literal symbols. Rather than utilizing working memory or inhibiting automatic prior referents, participants may engage set-shifting or cognitive shifting mechanisms during literal symbol processing to switch between a symbol's numerical and non-numerical (e.g., phonological) referents. Such mechanisms could be conceptually similar to language switching costs, which may lead to false generalizations across contexts (Oller & Ziahosseiny, 1970), or in which different contexts for learning and application require additional, task-specific cognitive processing (Grabner, Saalbach, & Eckstein, 2012; Spelke & Tsivkin, 2001).

In contrast to executive functioning, we found support for mapping performance as a mechanism that underlies literal symbol processing. Greater variability in mapping response time during training was associated with larger literal symbol processing costs. This suggests a ripple effect, in which weaker mapping reliability during training relates to a larger processing cost when using literal symbols in a subsequent task. This in turn suggests that literal symbol processing difficulty may not stem from trying to access literal symbol-referent connections in a mathematical task, but from the ability to form initial connections during the mapping process.

Taken together, the findings from this study further illuminate our understanding of literal symbol processing. When symbolizing numerical information, literal symbols show signatures of numerical magnitude processing, but with a processing cost in comparison to other symbols. However, this cost does not appear to stem from the need to inhibit prior referents or from working memory demands related to the associations between literal symbols and quantities. Rather, the processing cost may result from a difficulty in forming an initial mapping between literal symbols and related quantities, which in turn may affect subsequent task performance. Importantly, the literal symbol processing cost is also largely present at the individual level. There appear to be meaningful individual differences in performance when mapping literal symbols to quantity referents that relate to the ability to use literal symbols effectively as indicators of numerical magnitude.

4.4. Future directions

Findings of the present study suggests three avenues to further develop a theory of literal symbol processing. First, more work is needed on the reliability of the literal symbol processing cost and its related cognitive mechanisms. Future studies can seek to replicate this cost across samples of different ages or levels of experience with literal symbols, and examine test-retest reliability. Individual differences in the literal symbol processing cost do not appear to depend on executive function skills. However, particularly for visuo-spatial working memory, this relation needs further examination with a larger sample. Future studies should also include measures of cognitive shifting, in which learners shift their attention between different potential referents related to literal symbols; measures of proactive interference (e.g., retention of lists of learned words or items); and number-specific measures of inhibition (e.g., numerical Stroop task), which may relate to mathematical performance even when domain-general inhibition does not (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017). To further test the theory that pre-existing representations underlie the literal symbol processing cost, future studies could compare response times on same-different judgments for individual symbol-referent pairs that vary in phonological similarity (e.g., T – 3 versus Q – 9), or examine processing of literal symbols that have multiple pre-existing associations across contexts (e.g., K is associated with /k/ and also stands for the element Potassium). In addition, because literal symbols have pre-existing referents, they also have a verbal label (i.e., a name), while novel symbols do not. The need to retrieve a verbal label may hinder literal symbol processing (compared to novel symbols), or alternatively, their familiarity may facilitate performance, though that does not seem to be the case here. An intermediate condition with symbols that have labels but not strong prior referents, like shapes, may be a route to test the role of verbal labels.

Second, the present study suggests the need to characterize what it means to learn symbol-referent mappings efficiently. Symbol-referent training required participants to achieve a high level of accuracy, precluding an analysis of whether individual differences in mapping accuracy during training relate to the literal symbol processing cost. A measure of efficient mapping should include not only response time, variation in response time, and variation in accuracy, but a measure of monotonicity (e.g., increasing or decreasing response times) to capture learning over the course of training. The development of an index of these measures could be used to study not only literal symbol processing, but mapping in numerical cognition more generally (e.g., symbolic and non-symbolic representations).

Third, the present study examined mapping performance using one-to-one associations between literal symbols and their numerical referents. Due to a dearth of research on literal symbol processing, one-to-one associations between literal symbols and unique referents are a necessary and useful starting point to understand literal symbol processing, such as processing differences between these symbols and digits. Theories of literal symbol processing are in their infancy. As they develop, the notion of literal symbol-referent connections can and should expand to more closely align with how literal symbols are used in mathematics contexts. As two examples, studies should investigate multiple instances of forming and then breaking different associations between literal symbols and their numerical referents (e.g., $x = 3$, then $x = 6$, then $x = 2$), and examine literal symbols that represent a range of values (e.g., $a > 5$). However, without first building theory for foundational literal symbol processing, as offered here, investigations of more complex literal symbol processing are premature.

5. Conclusion

This study examined literal symbol processing and its potential mechanisms using numerical judgment tasks across three different symbol sets. All symbols elicited distance effects, signatures of numerical magnitude processing. There was a literal processing cost both as a group trend and for a majority of individual participants, whereby response times were longer for making numerical judgments with literal symbols. The ability to reliably map literal symbols to their numerical referents emerged as a potential mechanism contributing to literal symbol processing when making numerical judgments. Future research should examine the magnitude of this processing cost across different levels of mathematical expertise, and further examine candidate cognitive mechanisms that contribute to it. Doing so will build foundational knowledge that illuminates the difficulties associated with literal symbol processing, a necessary step to reduce the barriers to literal symbol processing in mathematics contexts.

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Declaration of competing interest

None.

Appendix A. Appendix

A.1. Analyses of error rate

There was a statistically significant main effect of symbol set [$F(2, 94) = 4.40$, $p = .02$, ${\eta_p}^2 = 0.087$]. Post-hoc Bonferroni-adjusted pairwise comparisons showed that error rate for literal symbol trials was 0.89 percentage points higher than for novel symbol trials, on average ($t = 2.82$, $p = .02$, 95% CI [0.105, 1.674]). There was a statistically significant main effect of distance [$F(1, 47) = 12.84$, $p = .001$, ${\eta_p}^2 = 0.215$], in which error rate was 0.87 percentage points greater on Near trials than Far trials ($t = 3.59$, $p = .001$, 95% CI [0.381, 1.355]). There was also a statistically significant interaction of symbol set and distance [$F(2, 94) = 4.73$, $p = .01$, ${\eta_p}^2 = 0.092$]. A simple effects analysis showed a distance effect for the number condition ($t = 2.79$, $p = .007$, 95% CI [0.365, 2.239]) and literal symbols condition ($t = 3.42$, $p = .001$, 95% CI [0.599, 2.309]), but not the novel symbols condition ($t = -0.46$, $p = .65$, 95% CI [−0.821, 0.517]). Fig. A1 displays the estimated means by symbol set and distance.

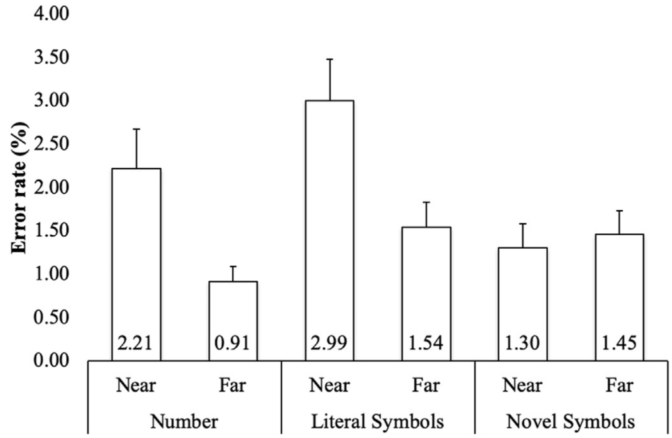

Fig. A1. Error rate by symbol set and distance. Error bars represent standard errors ($n = 48$).

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